Departure From Expectation

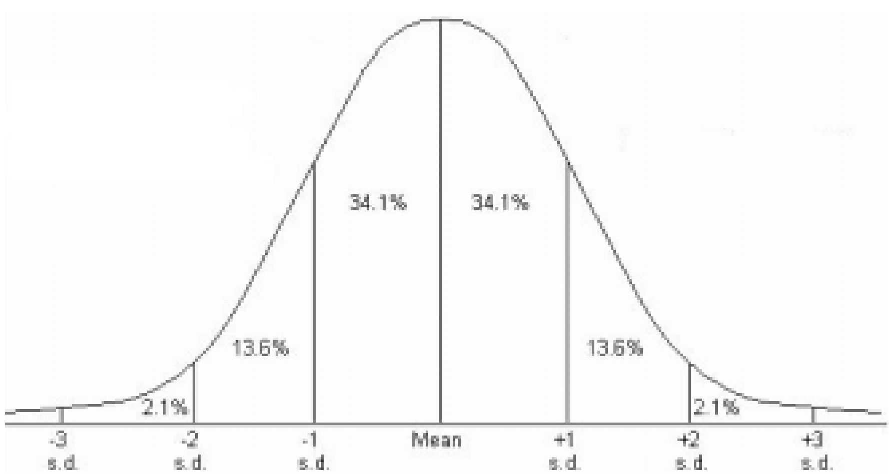
E Bruce Brooks
University of Massachusetts at Amherst

WSWG Note 56rev (10 Feb 1996)

We expect one thing, but we get another. There were 10 students in your class last year; this year, 15. There were 112 incidents of car theft last year, but this year, 121. Is this significant? Such questions come up all the time. They have a simple solution.

Normal Expectation

If occurrences diverge randomly from an expected value, the pattern of those departures is given by the normal distribution, whose curve looks like this:



Not so scary. All it means is that if things are varying randomly around a norm, 68·2% of all occurrences will fall within ±1 standard deviations (sd), and 95·4% within ±2 sd. If the matter is important, we want instead a 99% assurance that a given departure from expectation is meaningful, not random. That level is defined by ±2·54 sd.

For a given pair of E (expected) and A (actual) numbers, how do we figure the sd? The usual approximation formula[1] is:

$$(A - E) / \sqrt{E}$$

Which is simple enough. But to make it self-interpreting, so that the formula will give 1·00 when the 99% level of assurance is reached, we multiply by 0·39.[2]

[1]For the basic formula, see Paul G Hoel **Elementary Statistics** (2ed Wiley 1966) 103-106.
[2]Multiplying by 0·39 is the same as dividing by 2·54, the number of standard deviations.

The absolute value of (A – E) needs to be used in calculating, and any minus sign should be appended to the final result[3]. A further factor with human data is that once something unusual occurs, it is likely to repeat (a rare word in a text, a copycat crime). To compensate for this, we take the square root of the result. The final formula is:

$$S = \sqrt{[(0{\cdot}39)(A-E) / \sqrt{E}]}$$

This can be done in seconds on a hand calculator, as long as it has a square root key.

Practical Applications

1. Your class had 10 students last year; the expectation (E) is for 10 this year too. You actually (A) had 15. The significance (S) of this works out to

$$\sqrt{[(0{\cdot}39)(15-10) / \sqrt{10}]} = \sqrt{(0{\cdot}39)(5) / (3{\cdot}16)} = \sqrt{0{\cdot}61} = +0{\cdot}78, \textbf{ not} \text{ significant}$$

2. Over the 242 years covered by the Lǔ chronicle Chūn/Chyōu (CC), there are 524 military events, or 2·17 per average year. For the 18-year reign of Lǔ Hwán-gūng, we thus expect 39·06 military events; instead there are actually 16. The significance is:

$$\sqrt{[(0{\cdot}39)(16-39{\cdot}06) / \sqrt{39{\cdot}06}]} = \sqrt{(0{\cdot}39)(-23{\cdot}06) / (6{\cdot}25)} = \sqrt{-1{\cdot}44} = -1{\cdot}20, \textbf{ significant}$$

3. The average number of CC diplomatic events is 2·73 per year; for Hwán-gūng we expect 49·14 and actually get 58. The significance of this is:

$$\sqrt{[(0{\cdot}39)(58-49{\cdot}14) / \sqrt{49{\cdot}14}]} = \sqrt{(0{\cdot}39)(8{\cdot}86) / (7{\cdot}01)} = \sqrt{0{\cdot}49} = +0{\cdot}70, \textbf{ not} \text{ significant}$$

but this plus the preceding result suggests that the dip in military events may have had something to do with diplomatic efforts by Hwán-gūng, who went often to other states.

4. ευσεβεια "religion" occurs 15× in the New Testament (138,019 words), never in Paul, but 10× (A) in the post-Pauline Pastoral Epistles (9,488 words, so E = 1·03).

$$\sqrt{[(0{\cdot}39)(10-1{\cdot}03) / \sqrt{1{\cdot}03}]} = \sqrt{(0{\cdot}39)(8{\cdot}97) / (1{\cdot}01)} = \sqrt{3{\cdot}46} = +1{\cdot}86, \textbf{ significant}$$

This reflects the conventionalizing of Christian belief in the years after Paul.

5. The following are the actual crime statistics for a certain city in 1994 and 1995,[4] and their calculated significance:

	1994 (E)	1995 (A)	S	Interpretation
Murder	0	1	+0·62	not significant
Sexual Assault	55	47	−0·65	not significant
Assault/Battery	295	308	+0·54	not significant
Breaking/Entering	91	109	+0·86	**not yet** significant
Car Theft	112	121	+0·58	not significant
Vandalism	571	520	−0·91	**not yet** significant

Some of this should be watched by an alert Chief, though there is no present need to shift staff from one category to another. But there is an interesting social undercurrent: profitless crimes (vandalism) are down; profitable crimes (breaking/entering) are up.

S, as here presented, is for situations adequately described by one A and one E. Any real-life Chief would of course be tracking crimes over more than one year.

[3] A more elaborate way to say this is to multiply the final result by (A - E) / | A - E |.

[4] From the Daily Hampshire Gazette, 21 Feb 1996. As a matter of retrospective compassion, I do not here discuss the paper's own interpretation of these events.